Comparison of Credal Assignment Algorithms in Kinematic Data Tracking Context

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Abstract. This paper compares several assignment algorithms in a multi-target tracking context, namely: the optimal Global Nearest Neighbor algorithm (GNN) and a few based on belief functions. The robustness of the algorithms are tested in different situations, such as: nearby targets tracking, targets appearances management. It is shown that the algorithms performances are sensitive to some design parameters. It is shown that, for kinematic data based assignment problem, the credal assignment algorithms do not outperform the standard GNN algorithm.

Keywords: multi-target tracking, optimal assignment, credal assignment, appearance management.

1 Introduction

Multiple target tracking task consists of the estimation of some random targets state vectors, which are generally composed of kinematic data (e.g. position, velocity). Based on some measured data (e.g. position in x and y directions), the state estimation can be ensured by: Kalman filters, particles filters, Interacting Multiple Model algorithm which are used in this article and so on. Targets state estimation quality depends on how accurately the measured data are assigned to the tracked targets. In fact the assignment task is quite hard as far as the measured data are imperfect.

This paper focuses on distance optimization based assignment, where, the well known optimal Global Nearest Neighbor algorithm (GNN) [1] is compared with some, recently developed, equivalent credal solutions, namely: the works of Denoeux et al. [2], Mercier et al. [3], Fayad and Hamadeh [4] and Lauffenberger et al. [5]. Discussions with some other approaches are included [6].

This paper highlights drawbacks of turning distances into mass functions, in the credal algorithms. Simulation examples show the difficulties to correctly define the parameters of all the methods, including the appearances and disappearances management. In particular, it is shown that the performance criteria is linked to two distinct errors, namely: miss-assignment of two nearby targets and false decision about new targets. A method to define the most accurate mass function, allowing the credal algorithms to get the same performance as the GNN, is presented. It appears that in the described mono-sensor experiments

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based on kinematic data, credal assignment algorithms do not outperform the standard GNN algorithm.

In this paper, the problem of conflicting assignment situation and the proposed solutions are described in Section 2. A relation between the algorithms parameters is presented in Section 3. Some tests and results in tracking assignment context are depicted in Section 4. Section 5 concludes the paper.

2 Assignment Problem and Related Solutions

In multi target tracking contexts, updating the state estimations is much more complex than in a mono target framework. Indeed the first task is to assign the observations to the known objets.

Let us illustrate the problem in Fig. 1, where at a given time k, three targets $T = \{T_1, T_2, T_3\}$ are known and four observations $O = \{O_1, O_2, O_3, O_4\}$ are received. The question is: how to assign the observations to the known targets and taking into account the appearances and disappearances?

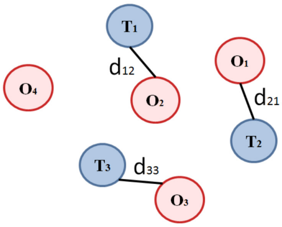


Fig. 1. Distance based assignment

Global Nearest Neighbor (GNN) solution: GNN algorithm is one of the firstly proposed solutions to the assignment problem in multi-target tracking context. It provides an optimal solution, in the sense where global distance between known targets and observations is minimized. Let $r_{i,j} \in \{0, 1\}$ be the relation that object T_i is associated or not associated with observation O_j ($r_{i,j} = 1$ means that observation O_j is assigned to object T_i, $r_{i,j} = 0$ otherwise). The objective function of such problem is formalized as follows:

$$min \sum_{i=1}^{n} \sum_{j=1}^{m} d_{i,j} r_{i,j} \ , \tag{1}$$

where,

$$\sum_{i=1}^{n} r_{i,j} = 1 \ , \tag{2}$$

$$\sum_{j=1}^{m} r_{i,j} \leq 1 \ , \tag{3}$$

where $d_{i,j}$ represents the Mahalanobis distance between the known target T_i and the observation O_j.

The generalized distances matrix $[d_{i,j}]$ for the example given in Fig. 1, can have the following form:

	O_1	O_2	O_3	O_4
T_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
T_2	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	$d_{2,4}$
T_3	$d_{3,1}$	$d_{3,2}$	$d_{3,3}$	$d_{3,4}$
NT_1	λ	∞	∞	∞
NT_2	∞	λ	∞	∞
NT_3	∞	∞	λ	∞
NT_4	∞	∞	∞	λ

When they are not assigned to existing targets, observations initiate new targets noted NT. If the probability p that an observation is generated by an existing target is known, the threshold λ can be derived from the χ^2 table as far as the Mahalanobis[1] distance follows a χ^2 distribution [7]:

$$P(d_{i,j} < \lambda) = p, \tag{4}$$

otherwise, it must be trained to lower the false decisions rate.

2.1 Denoeux et al.'s Solution [8]

In Denœux et al.'s approach as in most credal approaches, available evidence on the relation between objects T_i and O_j is assumed to be given for each couple (T_i, O_j) by a mass function $m_{i,j}$ expressed on the frame $\{0,1\}$ and calculated in the following manner:

$$\begin{cases} m_{i,j}(\{1\}) = \alpha_{i,j}, & \text{supporting } r_{i,j} = 1. \\ m_{i,j}(\{0\}) = \beta_{i,j}, & \text{supporting } r_{i,j} = 0. \\ m_{i,j}(\{0,1\}) = 1 - \alpha_{i,j} - \beta_{i,j}, & \text{ignorance on the assignment of } O_j \text{ to } T_i. \end{cases} \tag{5}$$

With R the set of all possible relations between objects T_i and O_j, $R_{i,j}$ denotes the set of relations matching object T_i with observation O_j:

$$R_{i,j} = \{r \in R | r_{i,j} = 1\}. \tag{6}$$

Each mass function $m_{i,j}$ is then extended to R by transferring masses $m_{i,j}(\{1\})$ to $R_{i,j}$, $m_{i,j}(\{0\})$ to $\overline{R_{i,j}}$ and $m_{i,j}(\{0,1\})$ to R. For all $r \in R$, associated plausibility function $Pl_{i,j}$ verifies:

$$Pl_{i,j}(\{r\}) = (1 - \beta_{i,j})^{r_{i,j}} (1 - \alpha_{i,j})^{1 - r_{i,j}}. \tag{7}$$

[1] For fair tests, all the algorithms are based on Mahalanobis distances.

After combining all the $m_{i,j}$ by Dempster's rule, the obtained global plausibility function Pl is shown to be proportional to the $Pl_{i,j}$ and given for all $r \in R$ by:

$$Pl(\{r\}) \propto \prod_{i,j}(1-\beta_{i,j})^{r_{i,j}}(1-\alpha_{i,j})^{1-r_{i,j}}. \tag{8}$$

Finally, the calculation of the logarithm function of (8), $\beta_{i,j}$ and $\alpha_{i,j}$ being all considered strictly lower than 1, allows the authors to express the search of the most plausible relation as a linear programming problem defined as follows with n objects T_i, m observations O_j and $w_{i,j} = \ln(1-\beta_{i,j}) - \ln(1-\alpha_{i,j})$:

$$max \sum_{i,j} w_{i,j} r_{i,j}, \quad i = \{1, ..., n\}, j = \{1, ..., m\}, \tag{9}$$

with

$$\sum_{i}^{n} r_{i,j} \leq 1 \ , \tag{10}$$

$$\sum_{j}^{m} r_{i,j} \leq 1 \ , \tag{11}$$

$$r_{i,j} \in \{0,1\}, \quad \forall i \in \{1, ..., n\}, \forall j \in \{1, ..., m\} \ . \tag{12}$$

This problem can be solved using Hungarian or Munkres algorithms [9]. More specifically, the authors also propose to solve an equivalent algorithm by considering, instead of (9), the following objective function:

$$max \sum_{i,j} w'_{i,j} r'_{i,j}, \quad i = \{1, ..., n\}, j = \{1, ..., m\}, \tag{13}$$

with $w'_{i,j} = max(0, w'_{i,j})$.

To experiment this algorithm with kinematic data based assignments, weights $\alpha_{i,j}$ and $\beta_{i,j}$ are computed in [8] as follows:

$$\begin{cases} m_{i,j}(\{1\}) = \alpha_{i,j} = \sigma \exp(-\gamma d_{i,j}) \\ m_{i,j}(\{0\}) = \beta_{i,j} = \sigma(1 - \exp(-\gamma d_{i,j})) \\ m_{i,j}(\{0,1\}) = 1 - \alpha_{i,j} - \beta_{i,j} = 1 - \sigma \end{cases} \tag{14}$$

where $d_{i,j}$ is the distance between object T_i and observation O_j and γ is a weighting parameter. Parameter σ is used to discount the information according to the sensor reliability [10].

In this article, all sensors have an equal perfect reliability, and so $\sigma = 0.9$. Moreover this parameter could be used in the same manner for all credal algorithms. On the contrary, parameter γ will be optimized. Although appealing, set of equations 14 remains empirical.

Mercier et al.'s solution [3]: mass functions in the works of Mercier et al. are calculated as in Equation (14), they are extended (vacuous extension [10]) to the frame of discernment $T^* = \{T_1, T_2, T_3, *\}$ or $O^* = \{O_1, O_2, O_3, *\}$ depending on if we want to associate the observations to the targets or the targets to the observations. Element $(*)$ models the non-detection or new target appearance. Once the mass functions are all expressed on a common frame of discernment, they are conjunctively combined [10] and then a mass function is obtained for each element O_j or T_i, according to the assignment point of view. Finally, The mass functions are transformed to pignistic probabilities [10]. The assignment decision is made by taking the maximum pignistic probabilities among the possible relations.

It is shown that this method is asymmetric when it comes to manage targets appearances and disappearances: assigning observations to targets is different than targets to observations. In this paper, only observations points of view are considered.

Lauffenberger et al.'s solution [5]: at the credal level, this method is almost similar to Mercier et al's method. To avoid the asymmetry problem, the authors, propose a different decision making strategy. For a given realization of distances, they perform the previous algorithm in both sides and obtain two pignistic probability matrices, which are not normalized since the weight on the empty set resulting from the conjunctive combination is isolated and used for a decision making purpose. A dual pignistic matrix is calculated by performing an element-wise product of calculated two pignistic probabilities matrices. The maximum pignistic probability is retained for each target (each row of the dual matrix), if this pignistic probability is greater than a given threshold, the target is associated with the column corresponding element, else it is considered as non-detected. The same procedure is performed for the observations (column elements of the dual matrix). The probabilities are also compared to the conflict weight generated by the conjunctive combination. If the conflict weight is greater that the dual matrix (rows/columns) probabilities, no assignment decision is made.

Fayad and Hamadeh's solution [4]: mass functions calculations in this method are different of the one adopted by the previous methods. For each observation O_j, in Fig. 1, for example, a mass function over the set of known targets $T^* = \{T_1, T_2, T_3, *\}$ is calculated. The element $(*)$ is a virtual target for which assigned observations are considered as new targets. Distances between known targets and each observation are sorted (minimum distance to maximum distance) and the mass function weights are calculated by inverting the distances and then normalizing the weighs in order to get a representative mass functions. Once mass functions of all observations are calculated, they are combined and expressed on the set of all possible hypotheses: $[\{(O_1, *), (O_2, *), (O_3, *), (O_4, *)\}, \{(O_1, T_1), (O_2, *), (O_3, *), (O_4, *)\}, ...]$. The combination is done by means of a cautious rule based on the "*min*" operator. This method becomes quickly intractable when the number of observations and/or targets gets over 3.

3 Relation between Parameters γ and λ

These two parameters can be optimized through a training. Moreover, if one is known the other can be deduced from it. To illustrate the necessity of choosing adequate parameters, let us consider that $d_{3,3}$, in Fig. 1, is equal to 1. The weights concerning the assignment of O_3 to T_3 for two different values of $\gamma = \{0.6, 0.8\}$ are: $\alpha_{i,j} = \exp(-0.6) = 0.55$ and $\beta_{i,j} = 1 - \exp(-0.6) = 0.45$. The parameter $\alpha_{i,j}$, in this case, is greater that $\beta_{i,j}$ so O_3 is associated to T_3. In another hand, if $\gamma = 0.8$, $\alpha_{i,j} = \exp(-0.8) = 0.45$ and $\beta_{i,j} = 1 - \exp(-0.8) = 0.55$, this means that T_3 is non-detected and O_3 is considered as a new target.

Fig. 2 represents the evolution of functions $\alpha_{i,j}$ and $\beta_{i,j}$.

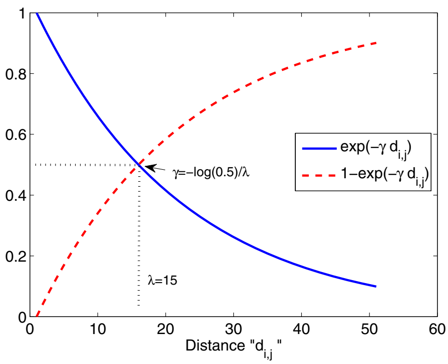


Fig. 2. Parameter "γ" determination

The fail-over distance (confirming/refuting the assignment) depends on the value given to γ. It can be chosen in such a way to get exactly the same fail-over distance as for the algorithm GNN, namely the parameter λ in Equation (4): it can be seen in Fig. 2 that the fail-over weight is given by the junction of the two curves ($\alpha_{i,j} = \beta_{i,j}$), if we want to impose λ as a fail-over distance, we just have to put $\exp(-\gamma\lambda) = 1 - \exp(-\gamma\lambda)$ and then deduce the value of γ using the following relation:

$$\gamma = \frac{-\log(0.5)}{\lambda}. \tag{15}$$

4 Assignment Tests in Tracking Context

Targets are evolving according to linear constant velocity models originally defined for aircrafts [1]:

$$x_i(k) = Ax_i(k) + Bu(k) + w(k) , \tag{16}$$

where:

$$A = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} (\Delta T)^2/2 & 0 \\ \Delta T & 0 \\ 0 & (\Delta T)^2/2 \\ 0 & \Delta T \end{bmatrix}, \tag{17}$$

where ΔT represents the sampling time and w represents a Gaussian state noise. Input matrix is represented by B, where B' is matrix B transpose. Vector $u = [a_x \quad a_y]^T$ in Equation (16) represents a given acceleration mode in x, y or both x and y directions.

Sensor measurements are modeled by:

$$O_i(k) = Hx_i(k) + v(k), \tag{18}$$

where,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{19}$$

and v a Gaussian measurement noise.

Let us start by giving a numerical example showing the effect of an arbitrary choice of the parameter γ, for example, in the concerned credal assignment methods.

Two time steps illustrating example: let $D(k) = [d_{i,j}]$ be a distances matrix at time step k, it is calculated based on 3 known targets $T_1(k)$, $T_2(k)$, $T_3(k)$ and 3 observations $O_1(k)$, $O_2(k)$, $O_3(k)$:

$$D(k) = \begin{bmatrix} 6.9 & 8.1 & 7.1 \\ 9.9 & 6.9 & 9.1 \\ 10.3 & 11.2 & 6.4 \end{bmatrix}. \tag{20}$$

The resolution of this matrix using GNN and Fayad's algorithms gives the following solution:

$$S_{G,F}(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{21}$$

The matrices $\alpha = [\alpha_{i,j}]$ and $\beta = [\beta_{i,j}]$, corresponding to the transformation of the distances into mass functions are given by:

$$\alpha(k) = \begin{bmatrix} 0.4514 & 0.4004 & 0.4425 \\ 0.3344 & 0.4514 & 0.3623 \\ 0.3213 & 0.2937 & 0.4746 \end{bmatrix}, \beta(k) = \begin{bmatrix} 0.4486 & 0.4996 & 0.4575 \\ 0.5656 & 0.4486 & 0.5377 \\ 0.5787 & 0.6063 & 0.4254 \end{bmatrix}, \tag{22}$$

For these matrices, the credal algorithms (except Lauffenberger et al.'s algorithm) gives the same solution as the GNN (see Equation (21)). Lauffenberger et al.'s algorithm gives the following solution:

$$S_L(k) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

This means that all the know targets are disappeared and all the observations are considered as new targets. This illustrates the limits of the algorithm in conflicting situation (nearby targets, given that the cross-distances are almost equal). This is due to the fact that the assignment decision is made based on the conflict generated by the mass functions combination, and when targets are close to each other, the conflict is high and then considerably influence the assignment decision.

At time step $k+1$, the measurements $O_1(k+1)$, $O_2(k+1)$, $O_3(k+1)$ of the same known targets are affected by the sensor noise, which leads to a different distance matrix $D(k+1)$:

$$D(k+1) = D(k) + noise = \begin{bmatrix} 7.8 & 9.4 & 8.5 \\ 10 & 7.8 & 11 \\ 10.2 & 12 & 7.9 \end{bmatrix}, \tag{24}$$

The obtained solution in the GNN and Fayad's algorithms is the same as in Equation (21). In order to get the other credal algorithms solutions, these distances are transformed into mass functions in the following matrices:

$$\alpha(k+1) = \begin{bmatrix} 0.4126 & 0.3516 & 0.3847 \\ 0.3311 & 0.4126 & 0.2996 \\ 0.3245 & 0.2711 & 0.4085 \end{bmatrix}, \beta(k+1) = \begin{bmatrix} 0.4874 & 0.5484 & 0.5153 \\ 0.5689 & 0.4874 & 0.6004 \\ 0.5755 & 0.6289 & 0.4915 \end{bmatrix}, \tag{25}$$

The algorithms depending on γ give the following solution:

$$S_{D,M,L}(k+1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{26}$$

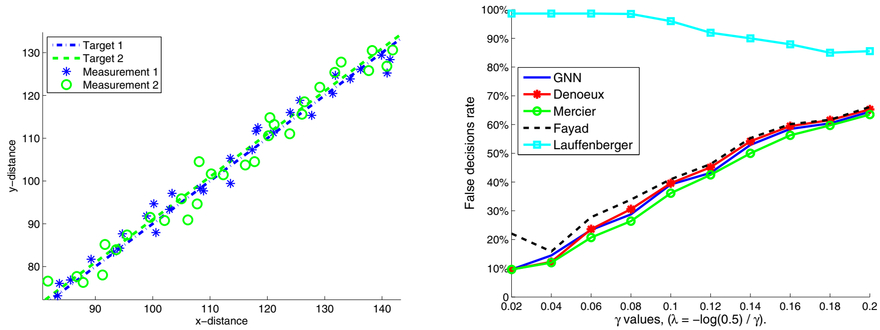
This solution means that all the known targets are not detected and all the acquired measurements are considered as new targets which is a false decision.

Let us now, consider the conflicting scenario of two nearby target, in (a) of Fig. 3 and compare the performances of the assignment algorithms which are given in (b) of the same figure.

The results in (b) Fig. 3 confirms the dependency of the algorithms on their respective parameters. In this simulation the parameters λ and γ are linked by the relation in Equation (15), therefore, λ depending algorithms and γ depending ones have almost the same performances. For a given value of their parameters, they supply the same performances, with the minimum error rate (10%) depending only on the scenario (noises and so on).

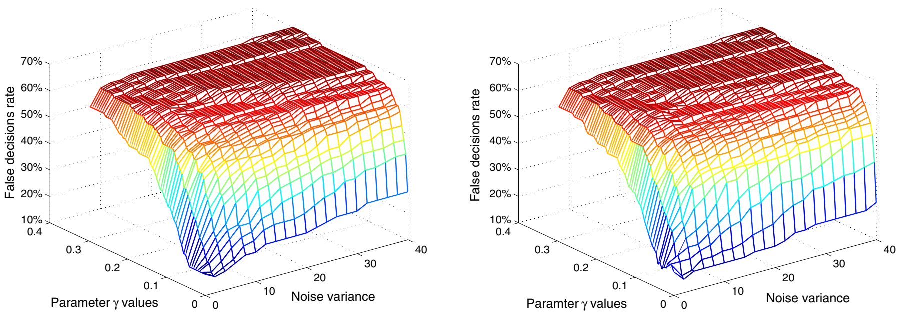
The results in Fig. 4 represent a robustness test of a λ depending algorithm, namely, GNN algorithm and a γ depending algorithm, namely, Denoeux's algorithm. The results shows that the algorithms are almost equivalent and similarly dependent on their respective parameters and sensor noise. Another robustness test is added in Fig. 5.

It can be seen in Fig. 5 that algorithms performances are sensitive and proportional to the modeling error which is simulated by state noise variation. In this simulation algorithms use the optimal values of their respective parameters.



(a) (b)

Fig. 3. (a): Conflicting scenario, (b): False assignments rates with the variation of the algorithms parameters



(a) (b)

Fig. 4. (a): GNN algorithm robustness test, (b): Denoeux's algorithm robustness test

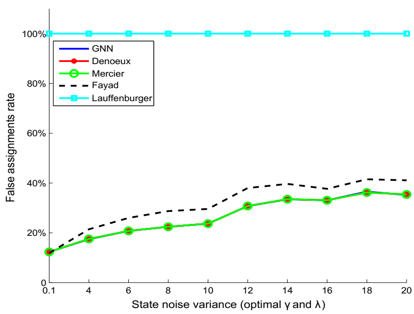


Fig. 5. Robustness against modeling error

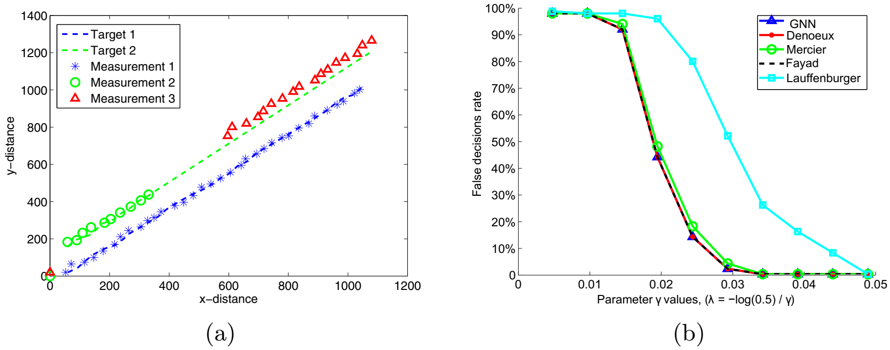


Fig. 6. (a): Target appearance scenario, (b): False decisions rate depending on the parameter λ for GNN and Fayad's algorithms and γ for Denoeux, Mercier and Lauffenberger's algorithms.

The following simulation (Fig. 6) confirms the almost equal performances on the second kind of errors about new targets appearances. This second simulation aims to calculate the false decisions rates, which means how often the newly detected target "Target 3", in (a) Fig. 6, is erroneously assigned to a previously non-detected one "Target 2".

It can be seen in (b) Fig. 6, that the false decisions rate depends on the parameter λ for the GNN and Fayad's algorithms, and depends on parameter γ for the other algorithms. The result shows that the algorithms reach equal performances for given values of λ and γ. When the probability p is known, λ can be determined according to Equation (2) and γ can be deduce using Equation (15).

A last simulation including the two precedent tests is added in the following. It tries to train the optimal parameters λ and γ on the scenario of Fig. 7, without any a priori knowledge.

Results of this simulation are obtained separately for λ depending algorithms (GNN and Fayad's algorithms) and γ depending algorithms (Denœux, Mercier and Lauffenberger's algorithms). They are depicted in Fig. 8.

As expected this last results show the necessity to adequately choose the algorithms parameters for a tracking assignment purpose. They also confirms that the trained optimal parameters $\lambda = 46$ and $\gamma = 0.015$ are linked by the relation of Equation (15) presented in Section 3, since $0.015 \simeq -log(0.5)/46$.

A final test is added to give an idea on the computational complexity of the studied algorithms. Computational times for an increasing number of observations are depicted in Fig. 9.

Test in Fig. 9 shows that Fayad's algorithm is the most computationally demanding. It is followed by Mercier's algorithm. GNN and Deneoux's algorithms seem to be the less complex algorithms.

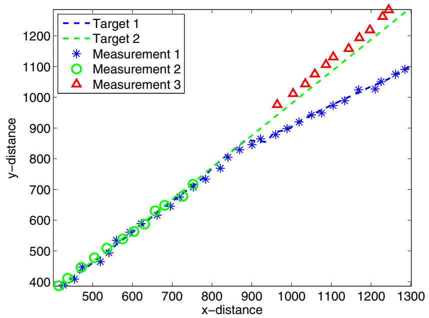
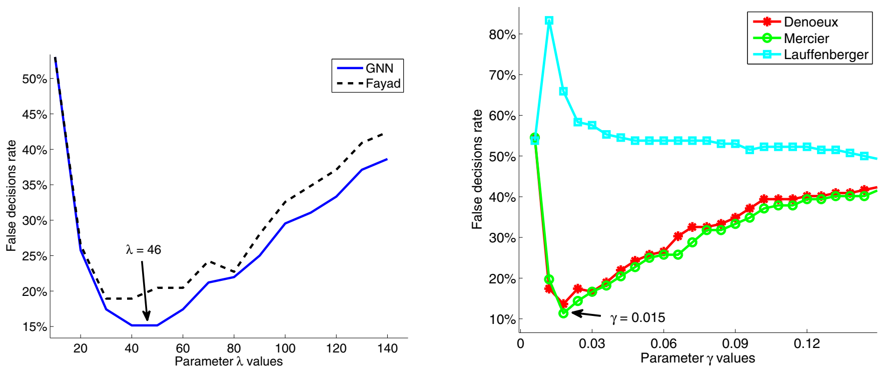


Fig. 7. Test on both false assignment and targets appearances management



(a) (b)

Fig. 8. (a): Performances of the algorithms depending on λ, (b): Performances of the algorithms depending on γ

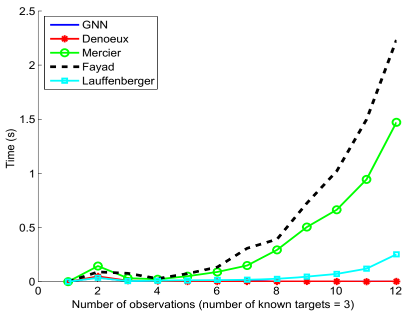


Fig. 9. Calculation time for an increasing number of data

5 Conclusion

This paper proposes a comparison study of various assignment algorithms in a context of multi-target tracking based on kinematic data. These algorithms depends on parameters that must be trained, otherwise, the performances are decreased. Contrarily to previous articles, it is shown here that the standard GNN algorithm with optimized parameters provides the same best performances than other algorithms. It is also less time-consuming. It is shown that there exists a relation between the optimized design parameters λ and γ.

It can be noticed that Lauffenberger's algorithm makes wrong decisions in conflicting scenarios. This is a priori due to the use of a decision making process based on conflict, where generated conflict in such situation is high and then refutes all assignments.

In the future, we will tackle the possible benefits of using belief functions in multi-sensors cases.

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