

# Tracking and Identification of Multiple targets

Samir Hachour, François Delmotte, Eric Lefèvre, David Mercier

Laboratoire de Génie Informatique et d'Automatique de l'Artois, EA 3926 LGI2A

[first.name.last.name@univ-artois.fr](mailto:first.name.last.name@univ-artois.fr)

**Abstract** – In this paper, we study the problem of joint tracking and classification of several targets at the same time. Targets are considered to be known and sufficiently separated so that they cannot be confused. Our goal is to propose a full methodology that is robust to missing information. The classical probabilistic approach with Bayesian tools is improved with belief functions. A simulation concerning the identification of go fast boats in a piracy problem shows that our approach improves previous results.

## 1. Introduction

In this article, we are interested in the problem of joint multi-target tracking and classification. This issue dates back to the 50's with the development of radars for control flight systems or air-defense systems ([3]). Solutions based on targets models and their kinematic data (position, velocity, etc.), already exist in the Bayesian framework ([4]) when there is enough statistical data. It is important to note that the task of classifying multiple targets simultaneously is much more complex than the single target classification one. Bayes rule resolves the former task by combining the measured likelihoods at a given time and the *a priori* classes probabilities. In multi-target case, the assignment of the measured likelihoods to the *a priori* probabilities requires another fundamental step which does not exist in the problem of single target tracking. It is the measures to targets assignment step, or the association step. The interested reader can refer to ([3] [1]).

In the single target framework, the works of Smets and Ristic ([8]) concerning the evidential classification had greatly improved the already existing results in the purely Bayesian framework.

This paper extends this methodology to the multi-target tracking and classification problem. This methodology can be applied to a large variety of situations when a large number of targets must be tracked and classified simultaneously in order to detect possible threats: pedestrian activities, aircrafts systems and so on. In this paper the application is the identification of go-fast boats used in modern piracy. We denote that the targets can appear or disappear at different unknown times. The only constraint we took is that models of evolution of the various targets are known in advance. Previous works carrying on multi-target credal classification already exist ([6] [9]) but without considering the targets tracking problem.

In our work the tracking step is ensured by Kalman filters and classic Bayesian IMM (Interacting Multiple Model), only the classification step is ensured by belief functions.

Due to a lack of space, the belief functions theory will not be exposed. Details of this theory can be found in ([7]).

In section 2, the targets evolution model and the classification problem are exposed. Section 3 details the IMM equations for each tracked target. The assignment step and targets appearance and disappearance problem are also treated, since we are carrying on a multi-target problem. Section 4 treats the classification problem and gives the Bayesian and the credal solutions. Finally in section 5, a comparison is performed to detect go fast boats used in modern piracy ([10]).

## 2. Targets evolution model

The evolution model is first formulated for one target and then generalized to multi-target case.

The target state vector at time  $k$  is denoted  $x_k \in \mathbb{R}^p$ , it represents the target kinematic data such as position, velocity, etc. One target can be classified into  $S$  different classes. The classes can be defined by:

$$C = \{c_1, c_2, \dots, c_S\}$$

In the context of maritime surveillance, class can be: a commercial boat, military boat or a small agile boat, etc.. Differentiation between classes is usually done using constraints on the velocity and acceleration of the target.

For simplicity, we assume that the target state evolves according to a linear model as follows:

$$x_k = F(c_i)x_{k-1} + Bu + w_k \quad (1)$$

where,  $F(c_i)$  is  $(p \times p)$  state matrix depending on the target class.  $B$  is the input matrix.  $u$  is a known input.  $w_k$  is a state white Gaussian noise with covariance matrix  $Q$ .

Also for simplicity, we consider that the measure at time  $k$  denoted  $z_k \in \mathbb{R}^q$ , is linearly dependent on the target state, according to the following equation:

$$z_k = Hx_k + v_k \quad (2)$$

where,  $H$  is the observation matrix of dimension  $(q \times q)$ .  $v_k$  is the measurement error, considered Gaussian noise with zero mean and covariance matrix  $R$ .

The optimal Bayesian estimation of the target class at time  $k$  requires the calculation of the probability density function  $p(x_k, c_i/Z_k)$  corresponding to the class  $c_i$ , where  $Z_k = \{z_1, z_2, \dots, z_k\}$  represents the cumulated measurement until the time  $k$ . The problem of estimating the targets state vector  $x_k$  is optimally resolved using the IMM algorithm ([2][3]).

### 3. Adaptive estimation of the targets motion

Over time a target can perform multiple movements, it can pass from a uniform to an accelerated one, or from a linear motion to a circular one, etc. Several works have been developed to represent the different movements that can make a target ([3]). Nowadays, there is no global model representing all possible modes of evolution of a target, that is why we adopt an adaptive approach to estimate the target motion. It consists on a multi-modal Kalman filters based estimation.

The idea consists on the use of as much Kalman filters as evolution modes of the target. A Bayesian process is used to switch between the different evolution modes, and the most likely Kalman filters is selected to estimate the target state. This is the key idea of the Interacting Multiple Model algorithm ([2] [3]).

The prediction and the update steps of the IMM algorithm are detailed in the following steps. For simplicity, the targets indices will be dropped out.

#### 3.1 IMM prediction step

1. Initialization step
  - Kalman filters state  $\hat{x}_i$  and covariance matrices  $P_i$  initialisation, with  $i = 1, \dots, r$ .
  - The different modes a priori probabilities  $\eta_i$ .
  - The  $(r \times r)$  IMM transition matrix  $\Pi$ . This matrix is used to mix the Kalman filters estimates.
2. Mixing step
  - Mixing probabilities calculation:

$$\mu_{ij}(k-1) = \frac{\Pi_{ij}\eta_i(k-1)}{C_j(k-1)} \quad (3)$$

where,  $\eta_i$  is the *a priori* probability that the target is in the mode  $i$  and the  $C_j$  is the probability that the target is in the mode  $j$  after interaction.

$$C_j(k-1) = \sum_{i=1}^r \Pi_{ij} \eta_i(k-1) \quad (4)$$

- Kalman state vectors and covariance matrices mixing:

$$\hat{x}_j^0(k-1/k-1) = \sum_{i=1}^r \mu_{ij}(k-1)\hat{x}_i(k-1/k-1) \quad (5)$$

$$P_j^0(k-1/k-1) = \sum_{i=1}^r \mu_{ij}(k-1)[P_i(k-1/k-1) + DP_{ij}(k-1)] \quad (6)$$

where,  $DP_{ij}(k-1)$  is calculated by:

$$DP_{ij}(k-1) = Dx_{ij}(k-1)Dx'_{ij}(k-1) \\ Dx_{ij}(k-1) = \hat{x}_i(k-1)D\hat{x}_i^0(k-1)$$

#### 3. Prediction step

- State  $\hat{x}_i$  and covariance matrices  $P_i$  prediction

$$\hat{x}_i(k/k-1) = F \hat{x}_i^0(k-1/k-1) + Bu_i \quad (7)$$

$$P_i(k/k-1) = F P_i^0(k-1/k-1)F' + Q_i \quad (8)$$

- Measurement  $\hat{z}$  and covariance matrices  $S_i$  prediction

$$\hat{z}_i(k/k-1) = H \hat{x}_i(k/k-1) \quad (9)$$

The innovation covariance matrix is given by:

$$S_i(k/k-1) = H P_i(k/k-1)H' + R \quad (10)$$

#### 4. Output (users)

$$\hat{z}(k/k-1) = \sum_{i=1}^r C_i \hat{z}_i(k/k-1) \quad (11)$$

$$S(k/k-1) = \sum_{i=1}^r C_i S_i(k/k-1) \quad (12)$$

where,  $\hat{z}(k/k-1)$  is the global predicted observation and  $S(k/k-1)$  is its prediction covariance matrix.

### 3.2 Observations to targets assignment

The IMM prediction step provides a set of predicted measurements  $\hat{z}^i$ ,  $i = \{1, 2, \dots, n\}$  for the  $n$  already known targets. These predicted measurements will be compared with a set of  $m$  real measurements  $\hat{z}^j$ ,  $j = \{1, 2, \dots, m\}$  at time  $k$ . An assignment problem will be resolved in such a way to efficiently recognize the origin of each real measurement:

- Assignment matrix  $M$  calculation:

$$M_{ij} = D_{ij}^2 + \ln[|S_i|] \quad (13)$$

where,

$$D_{ij}^2 = \tilde{z}'_{ij} S_i^{-1} \tilde{z}_{ij} \quad (14)$$

and

$$\tilde{z}_{ij} = \hat{z}^i - \hat{z}^j$$

- The assignment problem posed by the matrix  $M$  is solved using the auction algorithm ([3]) in such a manner to minimize the global distance between the predicted and the real measurements.

### 3.3 Targets appearance and disappearance management

The score function is calculated for each target in order to manage appearance or disappearance. It is a sequential likelihood ratio representing the quality of each target's track [3]. The test's steps are summarized as follows:

- hypotheses definition:  
 $H_0$ : hypothesis that the tracked target  $i$  is a false one.  
 $H_1$ : hypothesis that the tracked target  $i$  is a true one.
- the log-likelihood ratio  $L_i(k)$  of the target  $i$  at time  $k$  is sequentially calculated by:

$$L_i(k) = L_i(k-1) + \Delta L_i(k) \quad (16)$$

where,

$$\Delta L_i(k) = \begin{cases} \ln[1 - P_D] & i \text{ not detected at scan } k \\ \Delta L'_i(k) & i \text{ detected at scan } k \end{cases}$$

$$\Delta L'_i(k) = \ln \left[ \frac{P_D}{(2\pi)^{q/2} \beta_{FT} \sqrt{|S_i|}} \right] - \frac{d_i^2}{2} \quad (17)$$

where,  $P_D$  is the sensor detection probability and  $\beta_{FT}$  is the acceptable false targets density.

The distance between the target  $i$  and its assigned measurement  $d_i^2$  is calculated following (19).

- once the ratio is calculated it is compared to two thresholds:  
if  $L_i(k) \geq T_2$ , hypothesis  $H_1$  is accepted and the target  $i$  is confirmed.  
if  $L_i(k) \leq T_1$ , hypothesis  $H_0$  is accepted and the target  $i$  is deleted.  
if  $T_1 \leq L_i(k) \leq T_2$ , non of the hypotheses is accepted and the target continue to be tracked.  
The thresholds  $T_1$  and  $T_2$  are given by:

$$T_1 = \ln \left[ \frac{b}{1-a} \right], \quad T_2 = \ln \left[ \frac{1-b}{a} \right]$$

with,  $a$  a false target confirmation probability and  $b$  a true target deletion probability.

### 3.4 IMM update step

Each IMM updates its target state estimation with the measurement provided by the assignment step.

#### 1. Kalman filters likelihoods calculation

- Innovation calculation

$$\tilde{z}_i(k/k) = z(k) - H \hat{x}_i(k/k-1) \quad (18)$$

based on this innovation, a normalized Euclidean distance can be calculated

$$d_i^2 = \tilde{z}_i S_i^{-1} \tilde{z}_i \quad (19)$$

- Kalman filters likelihoods

$$\lambda_i = \frac{\exp [-d_i^2/2]}{\sqrt{(2\pi)^m |S_i|}} \quad (20)$$

where,  $m$  is the measurement dimension

#### 2. Kalman filters probabilities update:

$$\eta_i(k) = \lambda_i C_i(k-1)/C \quad (21)$$

where the normalizing constant  $C$  is given by:

$$C = \sum_{i=1}^r \lambda_i C_i(k-1)$$

#### 3. Kalman filters estimates update

- Gain calculation

$$K_i = P_i(k/k-1) H' [H P_i(k/k-1) H' + R]^{-1} \quad (22)$$

- Kalman filters covariance matrices update

$$P_i(k/k) = [I - K_i] P_i(k/k-1) \quad (23)$$

- Kalman state vectors update

$$\hat{x}_i(k/k) = \hat{x}_i(k/k-1) + K_i [z(k/k) - \hat{z}_i(k/k-1)] \quad (24)$$

#### 4. Global state estimate (for users)

$$\hat{x}(k/k-1) = \sum_{i=1}^r \eta_i(k) \hat{x}_i(k/k) \quad (25)$$

## 4. Target classification

Usually, the targets classification is performed using the target estimated kinematic data such as velocity and acceleration.

Knowing the different possible classes for a target, Smets and Ristic ([8]) proposed the use of as many IMM as classes, where, each IMM includes the different modes of each class.

Ristic *et al.* ([4]) thought that it is better to consolidate all the possible evolution modes in one IMM and retrieve the likelihood of each class by knowing their respective modes.

The following paragraph describes the Bayesian classification adopted by Ristic and *al.* ([4]) and the credal classification used in ([5]).

## 4.1 Bayesian classification

Let  $M_C = \{m_1, m_2, \dots, m_r\}$  the set of targets motion possible modes. We define the set of modes belonging to the class  $c_i$  by:

$M_{c_i} = \{m_1, m_2, \dots, m_{r_i}\}$ , where,  $r_i$  is the number of modes, with  $i = 1, \dots, n$

From the equations the IMM's update step, one can have respectively, the likelihood  $\lambda_j$  and the probability  $\mu_j$  of each evolution mode  $j$ .

The class likelihood  $\Lambda_i$ ,  $i = 1, \dots, S$  is obtained as follows:

$$\Lambda_i = \sum_{j=1}^{r_i} \mu'_j \lambda_j \quad j = 1, \dots, r_i \quad (26)$$

with:

$$\mu'_i = \frac{\mu_i}{\sum_{j=1}^{r_i} \mu_j}$$

The classes probabilities can be recursively obtained as follows:

$$P(c_i/Z_k) = \frac{\Lambda_i}{\sum_{j=1}^S \Lambda_j P(c_j/Z_{k-1})} P(c_i/Z_{k-1}) \quad (27)$$

where,  $P(c_i/Z_k)$  represents the *a priori* probability of the class  $c_i$ .

## 4.2 Classification with belief functions

When using belief functions, the IMM's provides the users with plausibilities  $Pl\{c_i\}$  instead of likelihoods ([5] [9]). Once the plausibility  $Pl\{c_i\}$  of each class  $c_i$  is obtained. we can calculate the mass functions on each element  $A \in 2^C$ :

$$m(A) = \prod_{c_i \in A} Pl\{c_i\} \prod_{c_i \in \bar{A}} (1 - Pl\{c_i\}) \quad (28)$$

In order to make a decision on the tracked target class, the mass functions are transformed to a pignistic probability using the following equation:

$$BetP(c_i) = \sum_{c_j \in A} \frac{1}{|A|} \frac{m(A)}{1 - m(\emptyset)} \quad (29)$$

where,  $m(\emptyset)$  is the belief mass on the empty set. Then the BetP functions are used as normal probabilities. This approach enables to convey uncertain or missing data the longest possible time through the algorithm. The whole approach with belief functions is called a credal classification, here it is compared to the Bayesian classification.

## 5 Two targets classification example

In this section, we present and compare the results of both Bayesian and credal classification of two maritime targets.

The expected classes of the targets are:

- Class 1: class of targets having a low maneuver capacities (e.g. cargo).
- Class 2: class of targets having a medium maneuver capacities (e.g. military boat).
- Class 3: class of targets having a high maneuver capacities (e.g. small agile boats called go-fast boats in modern piracy).

The state vector  $x = [x \ y \ \dot{x} \ \dot{y}]$  represents the position and the velocity on  $(x, y)$  directions, it is the same for both targets. The state vector evolves following the state model (1), where:

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$$

with  $T$  represents the sampling time.

The targets measurements are taken according to the measurement model (2), with:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The vector  $u = [a_x \ a_y]$  of state model in the case of our application represents the targets acceleration, it is considered as a deterministic input. The differences in the acceleration capabilities allow to distinguish between the targets different classes.

The acceleration limitations for the *a priori* known classes are expressed as follows:

$$L_i \leq |a_x \pm a_y| \leq L_i$$

where,  $L_i=0,1g, 0,3g, 0,5g$  respectively, for the classes  $c_1, c_2$  and  $c_3$ , with  $g = 9.81 \text{ m/s}^2$  is the gravitational acceleration.

The different acceleration modes and their distribution over the classes are illustrated in Fig.1.

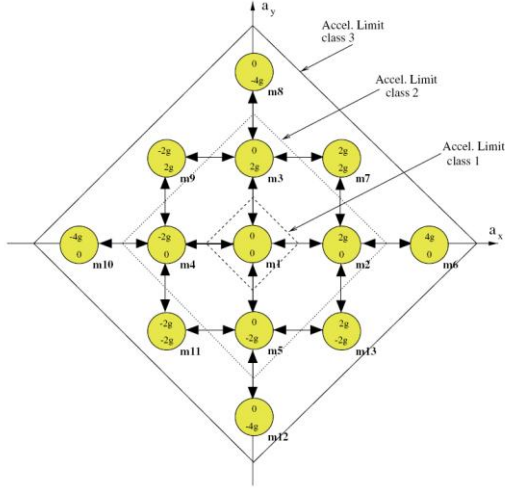


FIG. 1 : Different evolution modes

In our simulation, each target's IMM is composed about the 13 evolution modes of the figure FIG. 1. The modes are distributed over the three possible classes, as follows:

$M_{c_1} = [m_1]$ : modes of the class 1.

$M_{c_2} = [m_1, m_2, \dots, m_5]$ : modes of the class 2.

$M_{c_3} = [m_1, m_2, \dots, m_{13}]$ : modes of the class 3.

The non zero elements of the transition matrix  $\Pi$  of the IMM represent the interconnections between the modes (see FIG. 1). The diagonal elements of the transition matrix are equal to  $p = 0.9$  and the remaining  $1 - p$  is uniformly distributed on the non zero elements of the same row, in such a way to have a stochastic matrix.

### 5.1 Simulation results

The two dimensional trajectories of the targets are given in FIG. 2. As it can be seen, the trajectory of both targets consists of three constant velocity evolution segments and two maneuvers. The two targets do not disappear in this example.

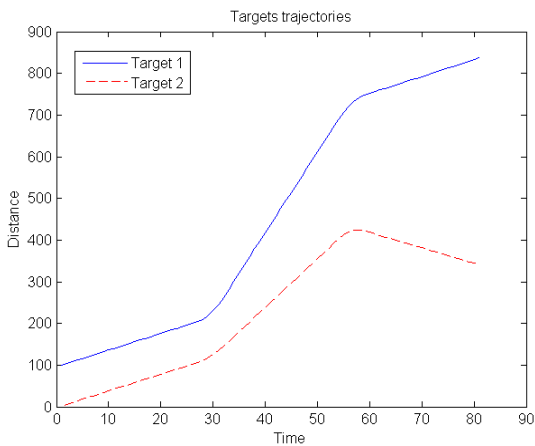


FIG. 2 : Targets trajectories

The first maneuver, for both targets, is performed between the 26<sup>th</sup> and 30<sup>th</sup> sampling times: The first target performs a sharp acceleration  $a_y = 0,4g$  in the  $y$  direction, while the second one performs a medium acceleration  $a_y = 0,2g$  in the same direction.

The second maneuver is performed between the 53<sup>th</sup> and 58<sup>th</sup> sampling times. In this time, both targets performs a sharp deceleration  $a_y = -0,4g$ , in  $y$  direction.

All the upcoming results represent the average over 25 simulations.

The Bayesian classification results of the two targets are respectively presented in the figures FIG. 3 and FIG. 4.

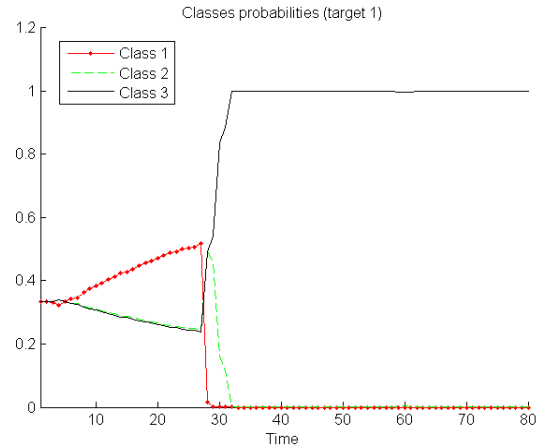


FIG. 3 : Bayesian classification of the target 1

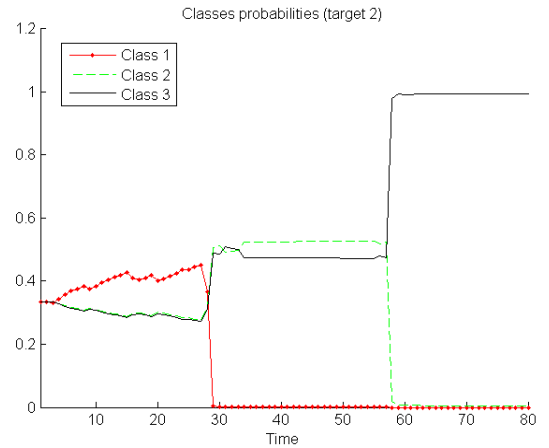


FIG. 4 : Bayesian classification of the target 2

Firstly, the targets are not adequately classified at the beginning. In fact, the classifier tends to increase the probability of the class 1, during the first movement step, where we are supposed to have a perfect doubt between the three classes. This is due to difference of the number of modes in each class, such as, more the class contains mode, less its likelihood is representative.

Finally, we notice that the two targets are correctly classified by the end of the simulation. The first target was in the class 1 and goes to the class 3 after its first sharp maneuver. The second target remains in doubt between the second and the third classes after its first medium maneuver. It goes to the class 3 after its second sharp maneuver.

Figure FIG. 5 and FIG. 6 represent the results of credal classification of the first and the second target respectively.

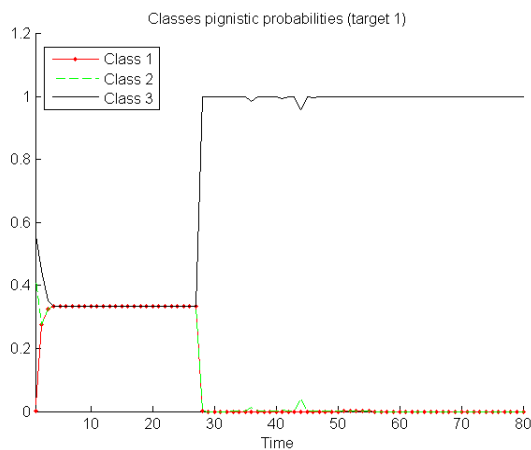


FIG. 5: credal classification of the target 1

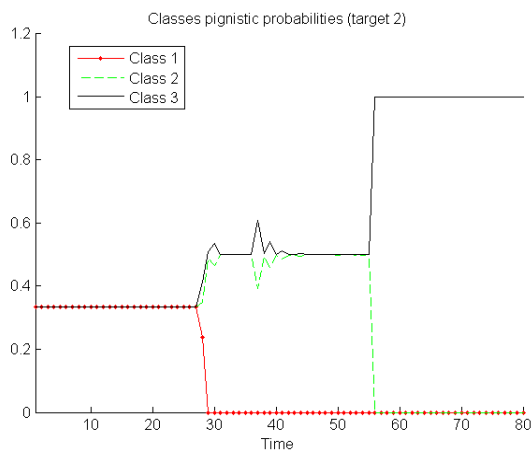


FIG. 6: credal classification of the target 2

Figures FIG. 5 and FIG. 6 show that the obtained credal classification results are better than those obtained using the Bayesian classification (FIG. 3 and FIG. 4) In fact, in the last credal results, we can see that the two targets are correctly classified during the three different steps of the movement. Credal classifier does not favor one class over another in case of doubt, as is the case for the Bayesian classification.

The ability of the credal formalism to associate one mass function to a set of classes in case of doubt, (e.g. associate one mass function to the set of the three classes in the first step of the targets movements in our example), allows to have three equivalent pignistic probabilities for the classes in doubt when taking a decision.

## 6. Conclusion

Multi-target classification is a fundamental problem, when it comes to classify multiple targets simultaneously. It is much more complex than the single target classification problem. Solutions already exist with a Bayesian framework ([3]) for air defense systems. Smets and Ristic ([8]) showed, in the single target framework, that the belief functions allow to greatly improve the Bayesian classification results. In this article, we mixed a target tracking step performed with classical IMM and Kalman filters and a classification step based on belief functions. We have adopted the Ristic and *al.* ([4]) idea for the tracking step, which consists on the use of only one IMM containing all the possible evolution modes for each target. Our results show that the purely Bayesian approach can be improved with belief functions. The whole methodology can be applied in various problems provided a model of evolutions of the various targets is known.

## References

- [1] A. Appriou. Multiple signal tracking processes. *Aerospace science and technology*, 1(3):165–178, 1997.
- [2] Y.Bar-Shalom and K.Birmiwal. Variable dimension filter for maneuvering target tracking. *Aerospace and Electronic Systems, IEEE Transactions on*, (5):621–629, 1982.
- [3] S.S. Blackman and R. Popoli. Design and analysis of modern tracking systems, volume 685. Artech House Norwood, MA, 1999.
- [4] B. Ristic, N. Gordon, and A. Bessell. On target classification using kinematic data. *Information Fusion*, 5(1):15–21, 2004.
- [5] B. Ristic and P. Smets. Belief function theory on the continuous space with an application to model based classification. In *Proceedings of Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU*, pages 4–9, 2004.
- [6] B. Ristic and P Smets. The TBM global distance measure for the association of uncertain combat ID declarations. *Information Fusion*, 7(1):276–284, 2006.
- [7] P. Smets and R. Kennes. The transferable belief model. *Artificial intelligence*, 66(2):191–234, 1994.
- [8] P. Smets and B. Ristic. Kalman filter and joint tracking and classification based on belief functions in the TBM framework. *Information Fusion*, 8(1):16–27, 2007.
- [9] N. Wartelle, F.Delmotte, and D.Gaquer. Classification de scènes à nombre d’objets inconnu. *Majestic (MANifestation des Jeunes Chercheurs STIC, novembre)*, Lorient, pages 22–24, 2006.
- [10] J.K.E. Tunaley, <http://www.london-research-and-development.com/GOFAST.pdf>