Extending the contextual discounting of a belief function thanks to its canonical disjunctive decomposition

David Mercier
Univ. Lille Nord de France, UArtois
EA 3926 LGI2A, France
Email: david.mercier@univ-artois.fr

***Abstract*—In this paper, the contextual discounting of a belief function is extended thanks to the canonical disjunctive decomposition of this belief function. A more general family of correction mechanisms allowing one to weaken the information provided by a source is then introduced, as well as the dual of this family, allowing one to strengthen a belief function.**

Keywords: Dempster-Shafer theory, belief functions, contextual discounting, disjunctive and conjunctive canonical decompositions.

I. Introduction

In the Dempster-Shafer theory of belief functions [2], [14], the reliability of a source of information is classically taken into account by the discounting operation [14, page 252], which transforms a belief function into a weaker, less informative one. This operation is usually important in information formation [1], [5], [6], [10], [11], [13], [20].

Introduced in [9], the contextual discounting is a refinement of the discounting operation. It takes into account the fact that the reliability of a source of information can be expected to depend on the true answer of the question of interest.

For instance, in medical diagnosis, depending on his/her specialty, experience or training, a physician may be more or less competent to diagnose some types of diseases. Likewise, in target recognition, a sensor may be more capable of recognizing some types of targets while being less effective for other types.

In this contextual model, the agent in charge of the fusion process or the decision making can hold a knowledge, regarding the reliability of a source of information, which depends on elements which form a partition (a coarsening) of the universe of discourse. For example, a sensor in charge of recognizing targets can have different reliabilities knowing that the target is a helicopter, an airplane or a rocket, but not reliabilities knowing that the target is a helicopter or a rocket, and a helicopter or an airplane.

In this paper, the contextual discounting operation of a belief function, thanks to its canonical disjunctive decomposition [3], is shown to be a particular case of a more general correction process [8] allowing the discounting of a belief function in a finer way. Moreover,the dual version of this correction mechanism, allowing one to reinforce a belief function, is also introduced.

To develop the justifications of these mechanisms, belief functions are interpreted as expressing weighted opinions, irrespective of any underlying probability distributions, and the Transferable Belief Model [16], [17], [19] is adopted.

This paper is organized as follows. Background material needed on belief functions is recalled in Section II, then a new family of correction mechanisms encompassing the contextual discounting in particular is introduced in Section III, and finally, Section IV concludes this paper.

II. Belief functions: basic concepts

A. Representing information

Let us consider an agent Ag in charge of making a decision regarding the answer to a given question Q of interest.

Let $\Omega = \{\omega_1, \dots, \omega_K\}$, called the *frame of discernment*, be the finite set containing the possible answers to question Q.

The information held by agent Ag regarding the answer to question Q can be quantified by a *basic belief assignment (BBA) or mass function* m^Ω_{Ag}, defined as a function from 2^Ω to $[0,1]$, and verifying:

$$\sum_{A\subseteq\Omega} m^\Omega_{Ag}(A) = 1\ . \tag{1}$$

Function m^Ω_{Ag} describes the state of knowledge of agent Ag regarding the answer to question Q belonging to Ω. By extension, it also represents an item of evidence that induces such a state of knowledge. The quantity $m^\Omega_{Ag}(A)$ is interpreted as the part of the unit mass allocated to the hypothesis: "the answer to question Q is in the subset A of Ω".

When there is no ambiguity, the full notation m^Ω_{Ag} will be simplified to m^Ω, or even m.

The following definitions are considered.

- A subset A of Ω such that $m(A) > 0$ is called a *focal element* of m.
- A BBA m with only one focal element A is said to be *categorical* and is denoted m_A; we thus have $m_A(A) = 1$.
- Total ignorance is represented by the BBA m_Ω, called the *vacuous belief function*.
- A BBA m is said to be:
  - *dogmatic* if $m(\Omega) = 0$;

– *non-dogmatic* if $m(\Omega) > 0$;
– *normal* if $m(\emptyset) = 0$;
– *subnormal* if $m(\emptyset) > 0$;
– *simple* if m has no more than two focal sets, Ω being included.

Finally, $\overline{m}$ denotes the *negation* of m [4], defined by $\overline{m}(A) = m(\overline{A})$, for all $A \subseteq \Omega$.

B. Combining pieces of information

Two BBAs m_1 and m_2 induced by distinct and reliable sources of information can be combined using the *conjunctive rule of combination (CRC)*, also referred to as the *unnormalized Dempster's rule of combination*, defined for all $A \subseteq \Omega$ by:

$$m_1 \circledcirc m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \ . \tag{2}$$

Alternatively, if we only know that at least one of the sources is reliable, BBAs m_1 and m_2 can be combined using the *disjunctive rule of combination (DRC)*, defined for all $A \subseteq \Omega$ by:

$$m_1 \circledcirc m_2(A) = \sum_{B \cup C = A} m_1(B) m_2(C) \ . \tag{3}$$

C. Marginalization and vacuous extension on a product space

A mass function defined on a product space $\Omega \times \Theta$ may be *marginalized* on Ω by transferring each mass $m^{\Omega\times\Theta}(B)$ for $B \subseteq \Omega \times \Theta$ to its projection on Ω:

$$m^{\Omega\times\Theta\downarrow\Omega}(A) = \sum_{\substack{B \subseteq \Omega\times\Theta, \\ \mathrm{Proj}(B\downarrow\Omega)=A}} m^{\Omega\times\Theta}(B), \tag{4}$$

for all $A \subseteq \Omega$ where $\mathrm{Proj}(B \downarrow \Omega)$ denotes the projection of B onto Ω.

Conversely, it is usually not possible to retrieve the original BBA $m^{\Omega\times\Theta}$ from its marginal $m^{\Omega\times\Theta\downarrow\Omega}$ on Ω. However, the *least committed*, or *least informative BBA* [15] such that its projection on Ω is $m^{\Omega\times\Theta\downarrow\Omega}$ may be computed. This defines the *vacuous extension* of m^{Ω} in the product space $\Omega \times \Theta$ [15], noted $m^{\Omega\uparrow\Omega\times\Theta}$, and given by:

$$m^{\Omega\uparrow\Omega\times\Theta}(B) = \begin{cases} m^{\Omega}(A) & \text{if } B = A \times \Theta, A \subseteq \Omega, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

D. Conditioning and ballooning extension on a product space

Conditional beliefs represent knowledge that is valid provided that an hypothesis is satisfied. Let m be a mass function and $B \subseteq \Omega$ an hypothesis; the *conditional belief function* $m[B]$ is given by:

$$m[B] = m \circledcirc m_B. \tag{6}$$

If $m^{\Omega\times\Theta}$ is defined on the product space $\Omega \times \Theta$, and θ is a subset of Θ, the conditional BBA $m^{\Omega}[\theta]$ is defined by combining $m^{\Omega\times\Theta}$ with $m_{\theta}^{\Theta\uparrow\Omega\times\Theta}$, and marginalizing the result on Ω:

$$m^{\Omega}[\theta] = \left(m^{\Omega\times\Theta} \circledcirc m_{\theta}^{\Theta\uparrow\Omega\times\Theta} \right)^{\downarrow\Omega}. \tag{7}$$

Assume now that $m^{\Omega}[\theta]$ represents the agent's beliefs on Ω conditionally on θ, i.e., in a context where θ holds. There are usually many BBAs on $\Omega \times \Theta$, whose conditioning on θ yields $m^{\Omega}[\theta]$. Among these, the least committed one is defined for all $A \subseteq \Omega$ by:

$$m^{\Omega}[\theta]^{\Uparrow\Omega\times\Theta}(A \times \theta \cup \Omega \times \overline{\theta}) = m^{\Omega}[\theta](A). \tag{8}$$

This operation is referred to as the *deconditioning* or *ballooning extension* [15] of $m^{\Omega}[\theta]$ on $\Omega \times \Theta$.

E. Discounting

When receiving a piece of information represented by a mass function m, agent Ag may have some doubts regarding the reliability of the source that provided this information. Such metaknowledge can be taken into account using the discounting operation introduced by Shafer [14, page 252], and defined by:

$$^{\alpha}m = (1-\alpha)m + \alpha\, m_{\Omega} \ , \tag{9}$$

where $\alpha \in [0, 1]$.

A discount rate α equal to 1, means that the source is not reliable and the piece of information it provides cannot be taken into account, so Ag's knowledge remains vacuous: $m^{\Omega}_{Ag} = {}^{1}m = m_{\Omega}$. On the contrary, a null discount rate indicates that the source is fully reliable and the piece of information is entirely accepted: $m^{\Omega}_{Ag} = {}^{0}m = m$. In practice, however, agent Ag usually does not know for sure whether the source is reliable or not, but has some degree of belief expressed by:

$$\begin{cases} m^{\mathcal{R}}_{Ag}(\{R\}) & = & 1-\alpha \\ m^{\mathcal{R}}_{Ag}(\mathcal{R}) & = & \alpha, \end{cases} \tag{10}$$

where $\mathcal{R} = \{R, NR\}$, R and NR standing, respectively, for *"the source is reliable"* and *"the source is not reliable"*. This formalization yields expression (9), as demonstrated by Smets in [15, Section 5.7].

The discounting operation (9) of a BBA m is also equivalent to the disjunctive combination (3) of m with the mass function m_0^{Ω} defined by:

$$m_0^{\Omega}(A) = \begin{cases} \beta & \text{if } A = \emptyset \\ \alpha & \text{if } A = \Omega \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

with $\alpha \in [0, 1]$ and $\beta = 1 - \alpha$.

Indeed:

$$m \circledcirc m_0^{\Omega}(A) = m(A) m_0^{\Omega}(\emptyset) = \beta m(A) = {}^{\alpha}m(A), \ \forall A \subset \Omega \,,$$

and

$$\begin{aligned} m \circledcirc m_0^{\Omega}(\Omega) &= m(\Omega) m_0^{\Omega}(\emptyset) + m_0^{\Omega}(\Omega) \textstyle\sum_{A\subseteq\Omega} m(A) \\ &= \beta m(\Omega) + \alpha = {}^{\alpha}m(\Omega) \ . \end{aligned}$$

F. Contextual Discounting based on a coarsening

Let $\Theta = \{\theta_1, \ldots, \theta_L\}$ be a coarsening of Ω, which means that $\theta_1, \ldots, \theta_L$ form a partition of Ω [14, chapter 6].

Unlike (10), in the contextual model, agent Ag is assumed to hold beliefs on the reliability of the source of information conditionally on each θ_ℓ, $\ell \in \{1, \ldots, L\}$:

$$\begin{cases} m^{\mathcal{R}}_{Ag}[\theta_\ell](\{R\}) & = & 1 - \alpha_\ell = \beta_\ell \\ m^{\mathcal{R}}_{Ag}[\theta_\ell](\mathcal{R}) & = & \alpha_\ell \ . \end{cases} \quad (12)$$

For all $\ell \in \{1, \ldots, L\}$, $\beta_\ell + \alpha_\ell = 1$, and β_ℓ represents the degree of belief that the source is reliable knowing that the true answer of the question of interest belongs to θ_ℓ.

In the same way as in the discounting operation (9), agent Ag considers that the source can be in two states: reliable or not reliable [9], [15]:

- If the source is reliable (state R), the information m^Ω_S it provides becomes Ag's knowledge. Formally, $m^\Omega_{Ag}[\{R\}] = m^\Omega_S$.
- If the source is not reliable (state NR), the information m^Ω_S it provides is discarded, and Ag remains in a state of ignorance: $m^\Omega_{Ag}[\{NR\}] = m_\Omega$.

The knowledge held by agent Ag, based on the information m^Ω_S from a source S as well as metaknowledge $m^{\mathcal{R}}_{Ag}$ concerning the reliability of the source can then be computed by:

- Deconditioning the L BBAs $m^{\mathcal{R}}_{Ag}[\theta_\ell]$ on the product space $\Omega \times \mathcal{R}$ using (8);
- Deconditioning $m^\Omega_{Ag}[\{R\}]$ on the same product space $\Omega \times \mathcal{R}$ using (8) as well;
- Combining them using the CRC (2);
- Marginalizing the result on Ω using (4).

Formally:

$$m^\Omega_{Ag}[m^\Omega_S, m^{\mathcal{R}}_{Ag}] = \left(\circledcirc_{\ell=1}^{L} m^{\mathcal{R}}_{Ag}[\theta_\ell]^{\Uparrow \Omega \times \mathcal{R}} \circledcirc m^\Omega_{Ag}[\{R\}]^{\Uparrow \Omega \times \mathcal{R}} \right)^{\downarrow \Omega}. \quad (13)$$

As shown in [9], the resulting BBA m^Ω_{Ag} , only depends on m_S and on the vector $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_L)$ of discount rates. It is then denoted by ${}^{\boldsymbol{\alpha}}_{\Theta}m$.

Proposition 1 ([9, Proposition 8]): The contextual discounting ${}^{\boldsymbol{\alpha}}_{\Theta}m$ on a coarsening Θ of a BBA m is equal to the disjunctive combination of m with a BBA m^Ω_0 such that:

$$m^\Omega_0 = m^\Omega_1 \circledcirc m^\Omega_2 \circledcirc \ldots \circledcirc m^\Omega_L \ , \quad (14)$$

where each m^Ω_ℓ, $\ell \in \{1, \ldots, L\}$, is defined by:

$$m^\Omega_\ell(A) = \begin{cases} \beta_\ell & \text{if } A = \emptyset \\ \alpha_\ell & \text{if } A = \theta_\ell \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

Remark 1: Two special cases of this discounting operation can be considered.

- If $\Theta = \{\Omega\}$ denotes the trivial partition of Ω in one class, combining m with m_0 defined by (11) is equivalent to combining m with m_0 defined by (14), so this contextual discounting operation is identical to the classical discounting operation.
- If $\Theta = \Omega$, the finest partition of Ω, this discounting is simply called contextual discounting and denoted ${}^{\boldsymbol{\alpha}}m$. It is defined by the disjunctive combination of m with the BBA $m^\Omega_1 \circledcirc m^\Omega_2 \circledcirc \ldots \circledcirc m^\Omega_K$, where each m^Ω_k, $k \in \{1, \ldots, K\}$ is defined by $m^\Omega_k(\emptyset) = \beta_k$ and $m^\Omega_k(\{\omega_k\}) = \alpha_k$.

G. Canonical conjunctive and disjunctive decompositions

In [18], extending the notion of separable BBA introduced by Shafer [14, chapter 4], Smets shows that each non-dogmatic BBA m can be uniquely decomposed into a conjunctive combination of *generalized simple BBAs (GSBBAs)*, denoted $A^{w(A)}$ with $A \subset \Omega$, and defined from 2^Ω to $\mathbb{R}$ by:

$$A^{w(A)}: \begin{array}{lll} \Omega & \mapsto & w(A) \\ A & \mapsto & 1 - w(A) \\ B & \mapsto & 0 \ , \ \forall B \in 2^\Omega \setminus \{A, \Omega\} \ , \end{array} \quad (16)$$

with $w(A) \in [0, \infty)$ (in fact $w(A) \in (0, \infty)$ as m is non-dogmatic). The function w: $2^\Omega \setminus \{\Omega\} \to (0, \infty)$ is another representation of a non dogmatic mass function and is called the conjunctive weight function. Let us also note that the higher is the weight $w(A)$, the higher is the incertitude on A, then a weight will be more precisely called an *uncertain weight*[1].

Every non-dogmatic BBA m can then be canonically decomposed into a conjunctive combination of GSBBAs:

$$m = \bigcirc\!\!\!\!\cap_{A \subset \Omega} A^{w(A)} \ . \quad (17)$$

In [3], Denœux introduces another decomposition: the canonical disjunctive decomposition of a subnormal BBA into *negative GSBBAs (NGSBBAs)*, denoted $A_{v(A)}$ with $A \supset \emptyset$, and defined from 2^Ω to $\mathbb{R}$ by:

$$A_{v(A)}: \begin{array}{lll} \emptyset & \mapsto & v(A) \\ A & \mapsto & 1 - v(A) \\ B & \mapsto & 0 \ , \ \forall B \in 2^\Omega \setminus \{\emptyset, A\} \ , \end{array} \quad (18)$$

with $v(A) \in (0, \infty)$.

Every subnormal BBA m can be canonically decomposed into a disjunctive combination of NGSBBAs:

$$m = \bigcirc\!\!\!\!\cup_{A \supset \emptyset} A_{v(A)} \ . \quad (19)$$

Indeed, as remarked in [3], the negation of m can also be conjunctively decomposed as soon as m is subnormal (in this case, $\overline{m}$ is non-dogmatic). Then:

$$\begin{array}{rrcl} & \overline{m} & = & \bigcirc\!\!\!\!\cap_{A \subset \Omega} A^{\overline{w}(A)} \\ \Rightarrow & m & = & \overline{\bigcirc\!\!\!\!\cap_{A \subset \Omega} A^{\overline{w}(A)}} \\ & & = & \bigcirc\!\!\!\!\cup_{A \subset \Omega} \overline{A^{\overline{w}(A)}} \\ & & = & \bigcirc\!\!\!\!\cup_{A \supset \emptyset} A_{\overline{w}(\overline{A})} \ . \end{array} \quad (20)$$

The relation between functions v and w is then $v(A) = \overline{w}(\overline{A})$ for all $A \neq \emptyset$.

[1] As remarked by Didier Dubois after the presentation of F. Pichon's PhD thesis [12].

III. Extending the contextual discounting

In this section, the contextual discounting operation is extended and shown to be a particular member a family of correction mechanisms based on the disjunctive decomposition of a subnormal BBA introduced by Denœux in [3].

According to the previous definitions (16) and (18), BBAs m_ℓ, $\ell \in \{1,\dots,L\}$, defined in (15) by $m_\ell(\emptyset) = \beta_\ell$ and $m_\ell(\theta_\ell) = \alpha_\ell$, can be denoted $\theta_{\ell_{\beta_\ell}}$ or θ_{β_ℓ} in a simple way.

From (14) and (19), the contextual discounting on a coarsening $\Theta = \{\theta_1,\dots,\theta_L\}$ of Ω of a subnormal BBA m is thus defined by:

$$\begin{aligned} {}^{\boldsymbol{\alpha}}_{\Theta}m &= m \circledcirc \theta_{\beta_1} \circledcirc \dots \circledcirc \theta_{\beta_L} \\ &= \bigcirc\!\!\!\!\!\cup_{A \supset \emptyset} A_{v(A)} \circledcirc \theta_{\beta_1} \circledcirc \dots \circledcirc \theta_{\beta_L} \ . \end{aligned}$$

In particular, as $A_{v_1(A)} \circledcirc A_{v_2(A)} = A_{v_1 v_2(A)}$ for all non-empty subset A of Ω:

- The classical discounting of a subnormal BBA $m = \bigcirc\!\!\!\!\!\cup_{A\supset\emptyset} A_{v(A)}$ is defined by:

$$^{\alpha}m = \Omega_{\beta v(\Omega)} \bigcirc\!\!\!\!\!\cup_{\Omega \supset A \supset \emptyset} A_{v(A)} \ ; \quad (21)$$

- The contextual discounting (cf. Remark 1) of a subnormal BBA $m = \bigcirc\!\!\!\!\!\cup_{A\supset\emptyset} A_{v(A)}$ is defined by:

$$^{\boldsymbol{\alpha}}m = \bigcirc\!\!\!\!\!\cup_{\omega_k \in \Omega} \{\omega_k\}_{\beta_k v(\{\omega_k\})} \bigcirc\!\!\!\!\!\cup_{A \subset \Omega, |A|>1} A_{v(A)} \ . \quad (22)$$

These discounting operations are then particular cases of a more general correction mechanism defined by:

$$^{\boldsymbol{\alpha}\cup}m = \bigcirc\!\!\!\!\!\cup_{A \supset \emptyset} A_{\beta_A v(A)}, \quad (23)$$

where $\beta_A \in [0,1]$ for all $A \neq \emptyset$ and $\boldsymbol{\alpha}$ is the vector $\{\alpha_A\}_{A \neq \emptyset}$.

In [9], the interpretation of each β_A has been given only in the case where the union of the subsets A forms a partition of Ω, β_A being interpreted as the degree of belief held by the agent regarding the fact that the source is reliable, knowing that the value searched belongs to A (cf Section II-F).

Instead of considering (12), let us now suppose that agent Ag holds beliefs regarding the reliability of the source, conditionally on each subset A of Ω:

$$\left\{ \begin{array}{lcl} m^{\mathcal{R}}_{Ag}[A](\{R\}) &=& 1-\alpha_A = \beta_A \\ m^{\mathcal{R}}_{Ag}[A](\mathcal{R}) &=& \alpha_A \ , \end{array} \right. \quad (24)$$

where $\alpha_A \in [0,1]$.

In the same way as in Section II-F, the knowledge held by agent Ag, based on the information m^{Ω}_S from a source and on metaknowledge $m^{\mathcal{R}}_{Ag}$ (24) regarding the reliability of this source, can be computed as follows:

$$\begin{aligned} & m^{\Omega}_{Ag}[m^{\Omega}_S, m^{\mathcal{R}}_{Ag}] \\ &= \left(\circledcirc_{A \subseteq \Omega} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}} \circledcirc m^{\Omega}_{Ag}[\{R\}]^{\Uparrow \Omega \times \mathcal{R}} \right)^{\downarrow \Omega}. \quad (25) \end{aligned}$$

Proposition 2: The BBA m^{Ω}_{Ag} resulting from (25) only depends on m^{Ω}_S and the vector $\boldsymbol{\alpha} = \{\alpha_A\}_{A \subseteq \Omega}$. The result is denoted ${}^{\boldsymbol{\alpha}}_{2^\Omega}m$ and is equal to the disjunctive combination of m^{Ω}_S with a BBA m^{Ω}_0 defined by:

$$m^{\Omega}_0(C) = \prod_{\cup A = C} \alpha_A \prod_{\cup B = \overline{C}} \beta_B, \quad \forall C \subseteq \Omega. \quad (26)$$

Proof: For each $A \subseteq \Omega$, the deconditioning of $m^{\mathcal{R}}_{Ag}[A]$ on $\Omega \times \mathcal{R}$ is given by:

$$\begin{aligned} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}}(A \times \{R\} \cup \overline{A} \times \mathcal{R}) &= \beta_A, \quad (27) \\ m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}}(\Omega \times \mathcal{R}) &= \alpha_A. \quad (28) \end{aligned}$$

With $A \neq B$:

$$\begin{aligned} & (A \times \{R\} \cup \overline{A} \times \mathcal{R}) \cap (B \times \{R\} \cup \overline{B} \times \mathcal{R}) \\ & \qquad = (A \cup B) \times \{R\} \cup \overline{(A \cup B)} \times \mathcal{R} \ . \end{aligned}$$

Then:

$$\begin{aligned} & \circledcirc_{A \subseteq \Omega} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}}(C \times \{R\} \cup \overline{C} \times \mathcal{R}) \\ & \qquad = \prod_{\cup D = \overline{C}} \alpha_D \prod_{\cup E = C} \beta_E, \quad \forall C \subseteq \Omega \ , \end{aligned}$$

or, by exchanging the roles of C and $\overline{C}$:

$$\begin{aligned} & \circledcirc_{A \subseteq \Omega} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}}(\overline{C} \times \{R\} \cup C \times \mathcal{R}) \\ & \qquad = \prod_{\cup D = C} \alpha_D \prod_{\cup E = \overline{C}} \beta_E, \quad \forall C \subseteq \Omega \ . \end{aligned}$$

It remains to combine conjunctively $m^{\Omega}_{Ag}[\{R\}]^{\Uparrow \Omega \times \mathcal{R}}$ and $\circledcirc_{A \subseteq \Omega} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}}$ which have focal sets of the form $B \times \{R\} \cup \Omega \times \{NR\}$ and $\overline{C} \times \{R\} \cup C \times \mathcal{R}$, respectively, with $B, C \subseteq \Omega$. The intersection of two such focal sets is:

$$\begin{aligned} & (\overline{C} \times \{R\} \cup C \times \mathcal{R}) \cap (B \times \{R\} \cup \Omega \times \{NR\}) \\ & \qquad = B \times \{R\} \cup C \times \{NR\} \ , \end{aligned}$$

and it can be obtained only for a particular choice of B and C. Then:

$$\begin{aligned} & \circledcirc_{A \subseteq \Omega} m^{\mathcal{R}}_{Ag}[A]^{\Uparrow \Omega \times \mathcal{R}} \circledcirc m^{\Omega}_{Ag}[\{R\}]^{\Uparrow \Omega \times \mathcal{R}}(B \times \{R\} \cup C \times \{NR\}) \\ & \qquad = \left[\prod_{\cup D = C} \alpha_D \prod_{\cup E = \overline{C}} \beta_E \right] m^{\Omega}_S(B) \ . \quad (29) \end{aligned}$$

Finally, the marginalization of this BBA on Ω is given for all subsets A of Ω, by:

$$^{\boldsymbol{\alpha}}m(A) = \sum_{B \cup C = A} \left[\prod_{\cup D = C} \alpha_D \prod_{\cup E = \overline{C}} \beta_E \right] m^{\Omega}_S(B) \ . \quad (30)$$

■

Let us note that the above proof has many similarities with proofs presented in [9, Sections A.1 and A.3].

As in the case of contextual discounting operations considered in Section II-F, the BBA m^{Ω}_0 defined in Proposition 2 admits a simple decomposition described in the following proposition.

Proposition 3: The BBA m^{Ω}_0 defined in Proposition 2 can be rewritten as:

$$m^{\Omega}_0 = \bigcirc\!\!\!\!\!\cup_{A \supset \emptyset} A_{\beta_A} \ . \quad (31)$$

Proof: Directly from (26) and the definition (3) of the DRC. ■

From (31), the contextual discounting ${}^{\boldsymbol{\alpha}}_{2^\Omega}m$ of a subnormal BBA $m = \circledcup_{A\supset\emptyset} A_{v(A)}$ is defined by:

$$\begin{array}{rcl} {}^{\boldsymbol{\alpha}}_{2^\Omega}m & = & \circledcup_{A\supset\emptyset} A_{v(A)} \circledcup_{A\supset\emptyset} A_{\beta_A} \\ & = & \circledcup_{A\supset\emptyset} A_{\beta_A v(A)} \\ & = & {}^{\boldsymbol{\alpha}\cup}m \ . \end{array} \quad (32)$$

Contextual discounting ${}^{\boldsymbol{\alpha}}_{2^\Omega}m$ is thus equivalent to correction mechanism ${}^{\boldsymbol{\alpha}\cup}m$ introduced in this section. Each coefficient β_A can then be interpreted as the degree of belief held by the agent Ag regarding the fact that the source is reliable knowing that the true answer to the question Q of interest belongs to A.

In a similar way, a correction mechanism for a non-dogmatic BBA m can be defined, from the conjunctive decomposition of m, by:

$${}^{\boldsymbol{\alpha}\cap}m = \circledcap_{A\subset\Omega} A^{\beta_A w(A)} \ ; \quad (33)$$

where $\forall A \subset \Omega, \beta_A \in [0,1]$, and $\boldsymbol{\alpha}$ is the vector $\{\alpha_A\}_{A\subset\Omega}$.

This process allows the reinforcement of a BBA m, as the smaller is the uncertain weight, the higher is the mass on A. It has been recently considered in [7] to combine partially non-distinct beliefs. This paper offers an interpretation for coefficients β_A, and the link between this mechanism and the contextual discounting is presented in the following.

Correction mechanisms ${}^{\boldsymbol{\alpha}\cap}m$ (23) and ${}^{\boldsymbol{\alpha}\cup}m$ (33) are related in the following way.

Let us consider a subnormal BBA m, $\overline{m}$ is then non-dogmatic:

$${}^{\boldsymbol{\alpha}\cap}\overline{m} = \circledcap_{A\subset\Omega} A^{\beta_A \overline{w}(A)} \ . \quad (34)$$

Then:

$$\begin{array}{rcl} \overline{{}^{\boldsymbol{\alpha}\cap}\overline{m}} & = & \overline{\circledcap_{A\subset\Omega} A^{\beta_A \overline{w}(A)}} \\ & = & \circledcup_{A\subset\Omega} \overline{A^{\beta_A \overline{w}(A)}} \\ & = & \circledcup_{A\supset\emptyset} A_{\beta_A \overline{w}(\overline{A})} \\ & = & \circledcup_{A\supset\emptyset} A_{\beta_A v(A)} \\ & = & {}^{\boldsymbol{\alpha}\cup}m \end{array} \quad (35)$$

These two correction mechanisms can thus be seen as belonging to a general family of correction mechanisms.

Acknowledgements

These works have been financed by the French region Nord-Pas de Calais under the project CISIT (Campus International pour la Sécurité et l'Intermodalité des Transports).

IV. Conclusion and Future Works

In this paper a family of correction mechanisms based on the canonical decompositions of belief function, encompassing in particular the contextual discounting, has been presented and justified. It allows one to discount or reinforce a belief function.

Future works will aim at testing it on real data. Likewise, it would also be interesting to automatically learn the coefficients of the correction mechanisms from data, as done for the classical and the contextual discounting operations [5], [9].

References

[1] I. Bloch. Defining belief functions using mathematical morphology - Applications to image fusion under imprecision. *International Journal of Approximate Reasoning, volume 48, pages 437–465, 2008.*

[2] A. Dempster. Upper and Lower Probabilities Induced by Multivalued Mapping. *Annals of Mathematical Statistics*, volume AMS-38, pages 325–339, 1967.

[3] T. Denœux. Conjunctive and Disjunctive Combination of Belief Functions Induced by Non Distinct Bodies of Evidence. *Artificial Intelligence*, volume 172, pages 234–264, 2008.

[4] D. Dubois and H. Prade. A set-theoretic view of belief functions: logical operations and approximations by fuzzy sets. *International Journal of General Systems*, volume 12, pages 193–226, 1986.

[5] Z. Elouedi, K. Mellouli and Ph. Smets. Assessing sensor reliability for multisensor data fusion with the transferable belief model. *IEEE Transactions on Systems, Man and Cybernetics B*, volume 34, pages 782–787, 2004.

[6] S. Fabre, A. Appriou and X. Briottet. Presentation and description of two classification methods using data fusion based on sensor management. *Information Fusion*, volume 2, pages 49–71, 2001.

[7] A. Kallel and S.L. Hégarat-Mascle, Combination of partially non-distinct beliefs: the cautious adaptive rule, *International Journal of Approximate Reasoning, accepted for publication.*

[8] D. Mercier, T. Denœux and M.-H. Masson. A parameterized family of belief functions correction mechanisms. In *Proceedings of IPMU'08*, L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (Ed.), Torremolinos (Malaga), pages 306-313, June 22-27, 2008.

[9] D. Mercier, B. Quost, T. Denœux. Refined modeling of sensor reliability in the belief function framework using contextual discounting. *Information Fusion*, volume 9, pages 246–258, 2008.

[10] D. Mercier, G. Cron, T. Denœux and M.-H. Masson. Decision fusion for postal address recognition using belief functions. *Expert Systems with Applications*, volume 36, issue 3, part 1, pages 5643–5653, 2009.

[11] N. Milisavljević, I. Bloch, S. van den Broek and M. Acheroy. Improving mine recognition through processing and Dempster-Shafer fusion of ground-penetrating radar data, *Pattern Recognition, volume 36, pages 1233–1250, 2003.*

[12] F. Pichon. Belief functions: canonical decompositions and combination rules, PhD Thesis, Université de Technologie de Compiègne, March, 2009.

[13] E. Ramasso, C. Panagiotakis, D. Pellerin and M. Rombaut. Human action recognition in videos based on the Transferable Belief Model - Application to athletics jumps. *Pattern Analysis & Applications, volume 11, issue 1, pages 1–19, 2008.*

[14] G. Shafer. A mathematical theory of evidence. Princeton University Press, Princeton, N.J., 1976.

[15] Ph. Smets. Belief functions: the disjunctive rule of combination and the generalized Bayesian theorem. *International Journal of Approximate Reasoning*, volume 9, pages 1–35, 1993.

[16] Ph. Smets and R. Kennes. The Transferable Belief Model. *Artificial Intelligence*, volume 66, pages 191–243, 1994.

[17] Ph. Smets. What is Dempster-Shafer's model? In *Advances in the Dempster-Shafer theory of evidence*, R. R. Yager, M. Fedrizzi and J. Kacprzyk (Ed.), Wiley, New-York, pages 5–34, 1994.

[18] Ph. Smets. The canonical decomposition of a weighted belief. In *Proceedings of the 14th International Joint Conference on Artificial Intelligence (IJCAI'95)*, Morgan Kaufman (Ed.), San Mateo, California, USA, pages 1896–1901, 1995.

[19] Ph. Smets. The Transferable Belief Model for quantified belief representation. In *Handbook of Defeasible reasoning and uncertainty management systems*, D. M. Gabbay and Ph. Smets (Ed.), Kluwer Academic Publishers, Dordrecht, The Netherlands, volume 1, pages 267–301, 1998.

[20] Ph. Smets. Data Fusion in the Transferable Belief Model. *Proceedings of the 3rd International Conference on Information Fusion, FUSION'2000, Paris, France, July 10-13, pages PS21–PS33, 2000.*